

EFFICIENCY OF THE CYCLOTRON MASER INSTABILITY AND OCCURRENCE PROBABILITY OF JOVIAN DECAMETER RADIO EMISSIONS

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Abstract

Within the framework of Io-controlled Jovian decametric radio emissions, we consider that electrons are accelerated in the neighbourhood of Io and follow an adiabatic motion along magnetic field lines carried by the satellite. The loss cone built up by electrons which have disappeared in Jupiter’s ionosphere constitutes the source of free energy needed by the cyclotron maser instability mechanism to produce the radiation. This theoretical mechanism is supposed to be at the origin of most planetary radio emissions. We calculate the maximum growth rate of the waves amplified by the cyclotron maser instability as a function of the joventric longitude of Io, in order to determine emission zones presenting a high occurrence probability. The results are compared to the occurrences observed for the sources in the CML–Io phase diagram.

1 Introduction

Long term ground observations of Jovian decametric emission (DAM) revealed that the occurrence probability of the radiation is depending on two fundamental parameters. The central meridian longitude (CML, System III) which is related to the rotating magnetic field and the orbital phase of the Io satellite [Bigg, 1964]. In the CML–Io phase plane we mainly distinguish four regions of enhanced occurrence, the so-called Io-controlled emission, which have been labeled “sources” Io–A, Io–B, Io–C and Io–D [Carr et al., 1983]. As shown in Figure 1, those sources have a symmetric occurrence with regard to the Jovian magnetic field axis (at CML of about 200°) combined with specific Io-phase positions, i.e. at about 90° and 240° . According to Dulk’s model [1967] the DAM emission is beamed into a thin conical sheet where the “sources” are associated to the edges of a

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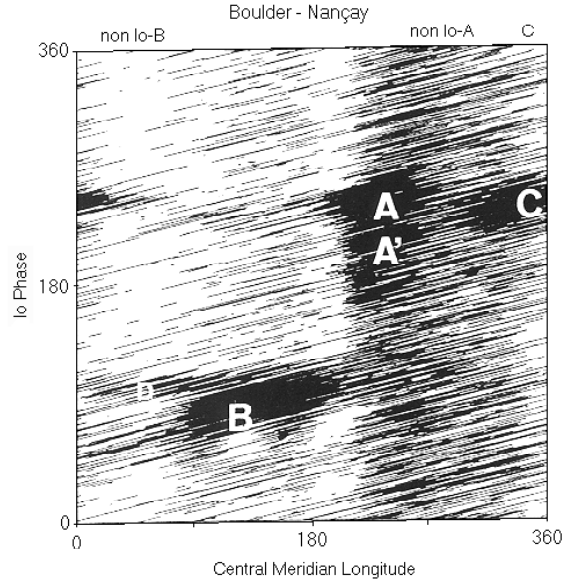


Figure 1: Occurrence probability of Jovian decameter radio emissions (as observed by ground-based radio stations) versus System III longitude (CML) and Io-phase.

hollow cone. In this case Io-A/Io-B and Io-C/Io-D belongs to two hollow cones located in the northern and southern hemispheres, respectively.

In order to interpret the control of the sources by the Jovian longitude, we derive the maximum growth rate of the waves amplified by the cyclotron maser instability, supposed to be at the origin of the radiation. We show how this mechanism seems to further some preferential Jovian longitudes for decameter radio emissions.

2 Theoretical maximum growth rate of the waves

2.1 Hypotheses

The cyclotron maser instability (CMI) is supposed to be the theoretical mechanism at the origin of several planetary radio emissions [Wu and Lee, 1979; Wu et al., 1982], in particular the Io-controlled decameter radiation of Jupiter (DAM). With the intent to determine the maximum growth rate of the waves produced by this mechanism, we have ventured several simplifying hypotheses. We have supposed that the radiation generated by the CMI is emitted within a hollow cone at the local gyrofrequency, the source of free energy needed by the CMI is a loss cone built up by electrons which disappear in Jupiter's ionosphere. We have also assumed that these electrons, accelerated in the vicinity of Io, follow an adiabatic motion along the magnetic field lines linked to Io. We make the assumption that the distribution function at Io of the electrons responsible for the radiation is Maxwellian.

When an electron comes into the Jovian ionosphere, its probability of disappearance is assumed to be proportional to the length of its actual path and to the ionosphere density.

Finally, taking into consideration the previous hypotheses, we derive the maximum growth rate ω_i of the CMI at several Jovian longitudes.

2.2 Cyclotron Maser Instability

The cyclotron maser instability is a resonant coupling between right-handed electromagnetic waves (relatively to the local magnetic field) and an electronic population forming a magnetized plasma. The source of free energy source needed by the CMI is contained in a positive gradient in perpendicular velocity $v_\perp$ of the electronic distribution function f ; i.e. $\partial f / \partial v_\perp$ must be positive in certain domains of the momentum space [Le Quéau et al., 1984a, 1984b; Ladreiter, 1991; Omid and Gurnett, 1982b; Omid et al., 1984]. The general equation of dispersion for a right-handed mode is:

$$k^2 - \frac{\omega^2}{c^2} - \frac{\mu_0 e^2}{4\pi m} \omega \iint \frac{\partial f / \partial v_\perp}{\omega - k_\parallel v_\parallel - \omega_c / \gamma} v_\perp^2 dv_\perp dv_\parallel = 0 \quad (1)$$

where $\mathbf{k}$ and ω refer to the wave-number and the frequency of the wave, $v_\perp$ and $v_\parallel$ are the components of the electronic velocity perpendicular and parallel relatively to the ambient magnetic field, ω_c denotes the local gyrofrequency, γ the Lorentz factor and m the electron mass. The frequency is a complex number:

$$\omega = \omega_r + i \omega_i, \quad \omega \in \mathbf{C}, \quad \omega_r, \omega_i \in \mathbf{R}. \quad (2)$$

The growth rate ω_i (imaginary part of the frequency) can be derived from Equation (1), ω_p denoting the plasma frequency:

$$\omega_i = \frac{\omega_p^2}{8} \int_0^{+\infty} v_\perp^2 dv_\perp \int_{-\infty}^{+\infty} \frac{\partial f}{\partial v_\perp} \delta \left(\omega_r - k_\parallel v_\parallel - \frac{\omega_c}{\gamma} \right) dv_\parallel \quad (3)$$

with the following normalization:

$$\iiint f(v_\parallel, v_\perp) d^3 \mathbf{v} = 1. \quad (4)$$

The Dirac function in Equation (3) is a resonant condition in the momentum space and the growth rate is an integral along this resonance condition.

2.3 Distribution function

We consider two distinct positions, at abscissa s and s' , upon a magnetic field line carried away by Io. We assume that there is no collision along the magnetic field line so the number of electrons is retained:

$$f(s, v, \alpha) v^2 \sin \alpha d\alpha dv ds dS = f(s', v', \alpha') v'^2 \sin \alpha' d\alpha' dv' ds' dS'. \quad (5)$$

α is the pitch angle, dS the section of an elementary flux tube and ds the movement along the field line.

The magnetic flux is always constant:

$$B \cdot dS = B' \cdot dS' . \quad (6)$$

The motion is supposed to be adiabatic:

$$\frac{v^2 \sin^2 \alpha}{B} = \frac{v'^2 \sin^2 \alpha'}{B'} , \quad (7)$$

and the kinetic energy is constant:

$$v = v' . \quad (8)$$

Then, we obtain:

$$\frac{ds}{\cos \alpha} = \frac{ds'}{\cos \alpha'} . \quad (9)$$

Replacing therefore Equations (6)–(9) in Equation (5) we notice that the distribution function along the magnetic field line is constant (in the position–momentum space):

$$f(s, v, \alpha) = f(s', v', \alpha') . \quad (10)$$

2.4 Probability of collision

The actual path dl of the electrons (with pitch angle α) during their adiabatic motion corresponding to a parallel movement ds is:

$$dl = \pm \frac{ds}{\cos \alpha} . \quad (11)$$

We suppose that the probability of collision between an individual electron and the ions in the Jovian ionosphere is directly proportional to the path followed by the electron and to the density of ions:

$$d\mathcal{P} = C \cdot e^{-z/H} \cdot dl , \quad (12)$$

supposing an exponential decay characterized by a scale height H . C is a constant depending on the cross-section of collision and z a Jovian altitude.

The total probability is derived from an integration of (12) along the full field line from Io ($+\infty$) to the mirror point (s_m) and then from the mirror point (s_m) to the current position (s):

$$\mathcal{P} = \int_{+\infty}^{s_m} \frac{d\mathcal{P}(s)}{ds} ds + \int_{s_m}^s \frac{d\mathcal{P}(s)}{ds} ds . \quad (13)$$

In the end, the probability of collision in the ionosphere for an up-going electron is:

$$\mathcal{P}(\alpha, s) = 2\pi C \sqrt{\frac{H \cdot B(s)}{g_{\parallel} \sin \lambda \sin^2 \alpha}} \exp \left[-\frac{\sin \lambda}{H g_{\parallel}} \left(B_s - \frac{B(s)}{\sin^2 \alpha} \right) \right] , \quad (14)$$

where $g_{\parallel}$ denotes the parallel magnetic field gradient, λ the inclination angle of the field line, B_s (resp. $B(s)$) the magnetic field strength at the mirror point (resp. current point).

2.5 Growth rate

Supposing that the distribution function at Io is a Maxwellian, we can derive from Equations (10) and (14) the distribution function $f(s, \alpha, v)$ at any position upon the magnetic field line carried away by Io:

$$f_{\text{down}}(s, \alpha, v) = \left(\frac{m}{2\pi kT} \right)^{3/2} e^{mv^2/2kT} \quad (15)$$

for $\pi/2 \leq \alpha \leq \pi$, i.e. down-going electrons, towards Jupiter, and

$$f_{\text{up}}(s, \alpha, v) = [1 - \mathcal{P}(s, \alpha)] \left(\frac{m}{2\pi kT} \right)^{3/2} \frac{e^{mv^2/2kT}}{\cos^2 \frac{\alpha_c}{2}} \quad (16)$$

for $\alpha_c \leq \alpha \leq \pi/2$, i.e. up-going electrons. In the previous equation α_c denotes the angle of the loss cone. The contours of this distribution function are plotted in Figure 2 for three distinct longitudes of Io: 120° and 300° at a position corresponding to 25 MHz (in the north) and 200° at 22 MHz (in the south).

The perpendicular gradient is easily obtained:

$$G_{\perp} = \frac{\partial f}{\partial v_{\perp}} = \frac{\cos \alpha}{v} \frac{\partial f}{\partial \alpha} + \sin \alpha \frac{\partial f}{\partial v}. \quad (17)$$

It is plotted in Figure 2 for the same longitudes of Io. It is worth noticing that the perpendicular gradient $G_{\perp}$ is positive only in a very thin region near the edge of the loss cone (of angle α_c).

In the end, replacing Equation (17) in Equation (3), it is possible to derive the maximum growth rate ω_i of the CMI from an integration along the resonance curve which can be approximated to a circle in the plane $(v_{\parallel}, v_{\perp})$:

$$\omega_i = \frac{\omega_p^2 c^2}{8\omega_c} \Xi^2 \int_0^{\pi} G_{\perp}(\Xi, \theta) \sin^2 \theta d\theta. \quad (18)$$

The radius of the resonance circle is:

$$\Xi = c \left[\frac{k_{\parallel}^2 c^2}{\omega_c^2} + 2 \left(1 - \frac{\omega}{\omega_c} \right) \right]^{1/2} \quad (19)$$

and its centre is located at the point $v_{\perp} = 0$, $v_{\parallel} = v_0$ (see Figure 3) with:

$$v_0 = \frac{k_{\parallel} c^2}{\omega_c^2}. \quad (20)$$

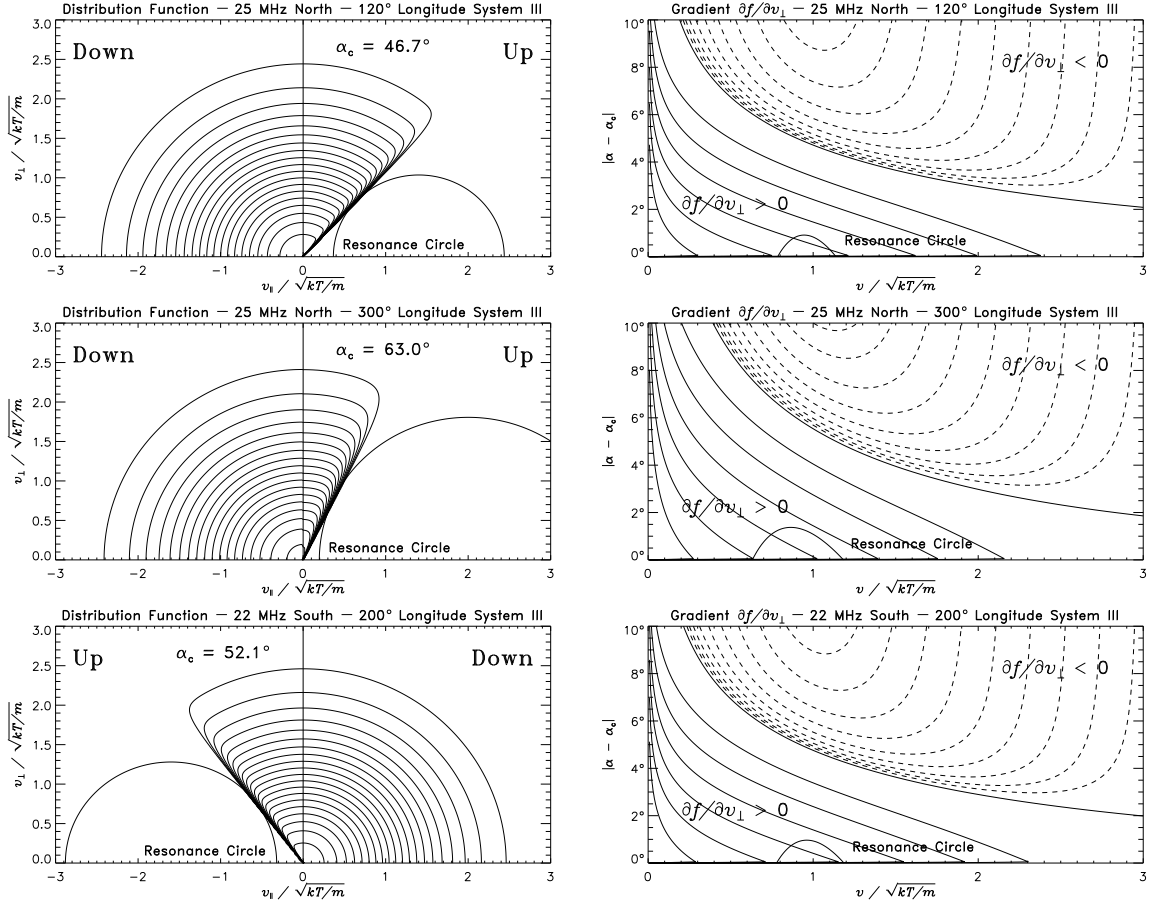


Figure 2: Left panels: contours of the electronic distribution function f at the point of the magnetic field line where the gyrofrequency is 25 MHz or 22 MHz. It is plotted for three different longitudes (of System III). Right panels: contours of the perpendicular gradient of the distribution function $\partial f / \partial v_{\perp}$ near the edge of the loss cone. The latter is characterized by its angle α_c . The optimum resonance circle defined in Sect. 2.6 is also displayed.

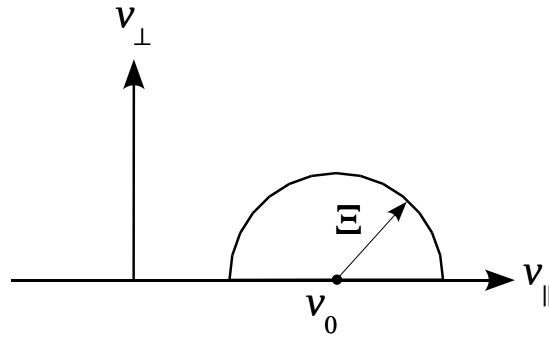


Figure 3: Resonance curve in the momentum space.

2.6 Optimum resonance circle

We assume that the waves having a strong occurrence probability are those produced with a maximum growth rate. We determine numerically, at a given frequency of emission (25 MHz, resp. 22 MHz, for the radiation escaping from the northern, resp. southern, hemisphere), the position of the resonance circle for which the growth rate ω_i is positive and maximum, for each position of the magnetic field line intersecting Io and characterized by its longitude. As displayed in Figure 2, the particular resonance circle leading to this maximum growth rate is nearly tangent to the edge of the loss cone of the distribution function and penetrates just in the region where $\partial f / \partial v_{\perp} > 0$.

3 Results

3.1 Growth rate versus System III longitude

In Figure 4, the maximum growth rate (actually a dimensionless normalized growth rate $\omega_i \omega_c kT / \omega_{p0}^2 mc^2$) is plotted as a function of the Jovian longitude of the magnetic field line along which the radio emission is supposed to occur. The calculation has been made for several frequencies: 25, 20, 15 and 12 MHz (resp. 22, 18, 15 and 12 MHz) for the northern (resp. southern) hemisphere. The curves in Figure 4 present a maximum at about 130° for the northern hemisphere and close to 200° for the southern hemisphere. This maximum value of ω_i as a function of the jovicentric longitude can be interpreted as the location of an active longitude where the emission mechanism is the most effective. The modulation of ω_i in longitude defines the expansion of the efficiency domain. This modulation, very pronounced beyond 20 MHz, fades very quickly when the frequency decreases and practically disappears under 12 MHz.

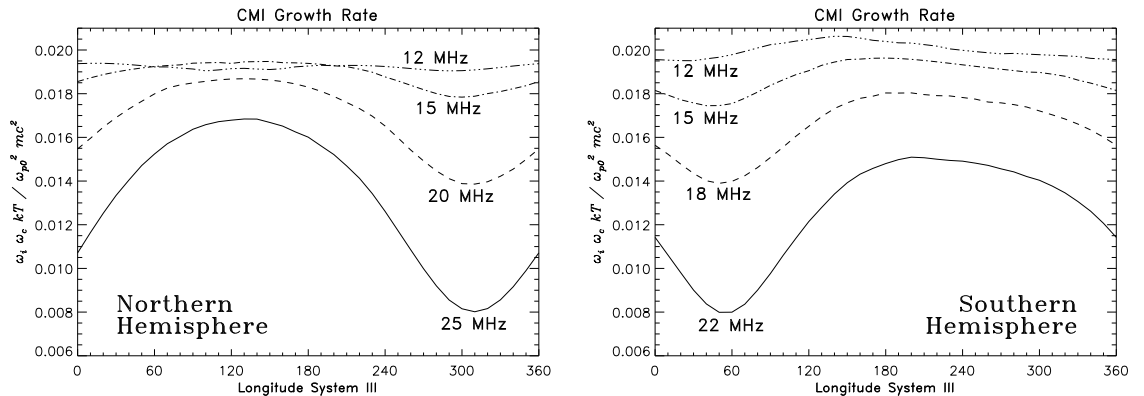


Figure 4: Growth rate of the cyclotron maser instability versus Jovian longitude (System III).

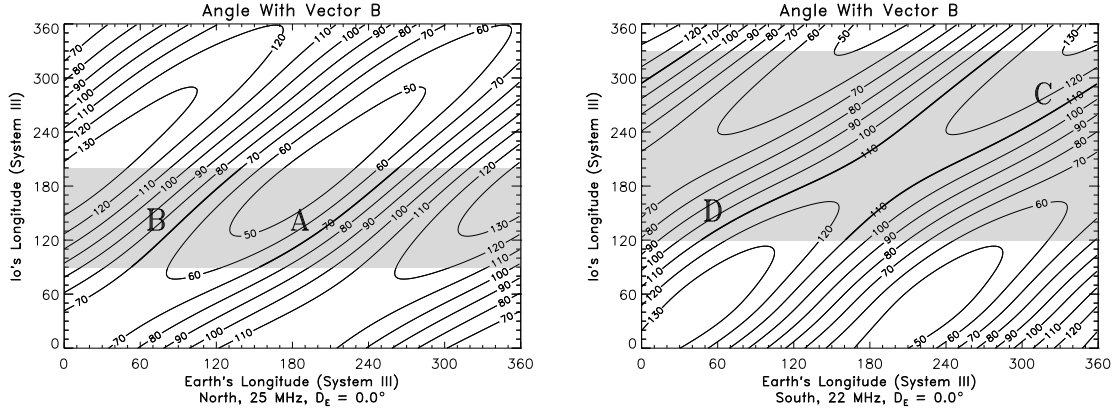


Figure 5: Contours of the angle between the local magnetic field and the direction of the observer (located at the Earth). An active longitude range is deduced from the curves in Figure 4 and displayed in grey. The thick segments correspond to emissions with a high probability of occurrence.

3.2 Source location

Figure 5 displays the contours of the angle between the direction of the observer (located at the Earth) and the direction of the local magnetic field, at the place where the emission occurs. The abscissa is the jovicentric longitude (System III) of the Earth and the ordinate is the longitude of Io. For both hemispheres, we have assumed $D_E = 0^\circ$. The grey area represents a certain domain of active longitude, carried away by Jupiter, and deduced from the curves in Figure 4. Two conditions are needed for the radio emission to be observed: (i) the magnetic field line connected to Io must come across the active domain of the CMI linked to Jupiter, and (ii) in the same time the observer must be in the emission cone of the radiation. The first condition corresponds to the grey area displayed in Figure 4. On the other hand, the emission is supposed to occur in a hollow cone of constant angle (we have chosen 70° for the north and 110° for the south) round the magnetic field so that the second condition corresponds to the contours in Figure 4. Thus both conditions are simultaneously fulfilled for the segments of the contours 70° (or 110°) which are located in the grey areas and marked by thick lines. Two such segments appear in each hemisphere.

In order to compare the theoretical zones of high emission probability to the observations, in particular to the occurrence diagram (see Figure 1), it is relevant to use a representation involving the phase of Io rather than its longitude. Thus in Figure 6 the theoretical “sources” are displayed in a CML–Io phase diagram. The comparison with Figure 1 is immediate and we can attempt to assimilate the segments of theoretical sources to the so-called Io–A, Io–B, Io–C and Io–D “sources”.

4 Conclusion

Making the assumption that electrons follow an adiabatic motion along magnetic field lines carried away by Io, and that the probability of collision is proportional to the length of the electronic path and to the ionospheric density, we show that the growth rate of the

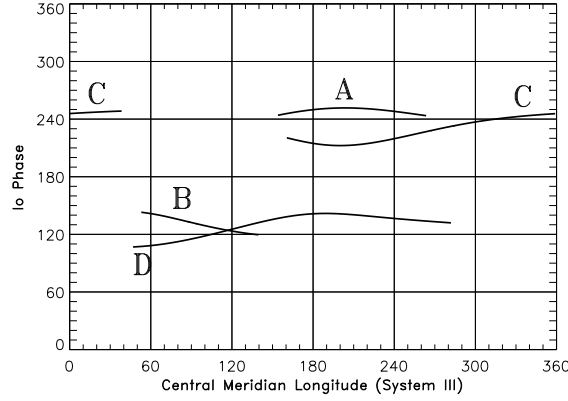


Figure 6: Curves of high occurrence probability versus jovicentric longitude (CML) and Io-phase.

CMI is larger for a certain range of Jovian longitude: (i) about $\text{CML} = 130^\circ$ in northern hemisphere, (ii) about $\text{CML} = 200^\circ$ in southern hemisphere. Supposing that the radio emission is produced within a hollow cone, we find zones of high emission probability in the CML–Io phase diagram presenting a certain agreement with the observed Io-controlled sources.

The agreement between theoretical and observed sources is not complete and the present model can be improved: (i) the scale height of the ionosphere may dependent on the local time, (ii) the electronic distribution function at Io is certainly not a Maxwellian, (iii) the emission cone may be non-axisymmetric or more complex. However an active range of Jovian longitude controls part of the Io–DAM emission above 15 MHz and subsides at lower frequencies.

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